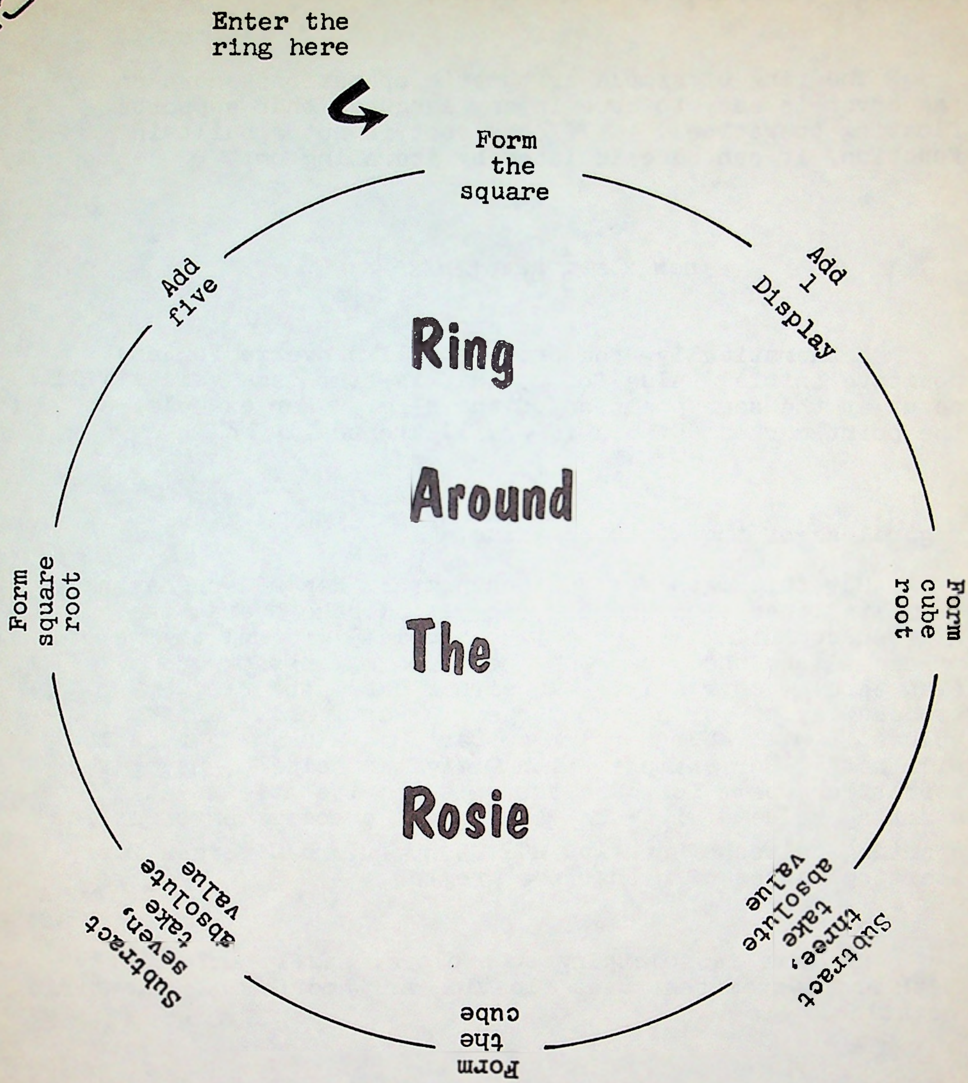
$2.50

63

Ring Around The Rosie



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Popular Computing

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Rosie's Ring—A Study In Convergence

The ring of simple arithmetic operations shown on the cover is easy to code in any language that supports floating operations. If cube root is not a built-in function, it can be calculated by iterating on:

$$\text{new } x = \frac{1}{3}\left[2x + n/x^2\right]$$

Mathematically, the process will converge for any positive initial value for n; that is, the same values will occur at the same position of the ring. For example, at the point marked "Display," n will eventually be

57.5806879155...

regardless of the starting value.

But this mathematical convergence depends on having unlimited precision and accuracy in all the operations. A given computing system may have only 9 decimal digits of precision and the cube root operation may not produce even those 9 correctly. In either case, the ring process may or may not converge. For small starting values it will, but for large starting values it probably will not. For example, with 9 digit precision, using the iterative scheme for cube root with 18 iterations and a starting value of $x_0 = n/3$, the whole process converges for starting values of n below 242.0903..., and diverges for starting values of n that are greater.

So here is something to explore, easily and quickly with any system that uses floating arithmetic and scientific notation.

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Program the ring of operations and see what happens when the number 10 is entered (it should converge under almost any conditions). Then try n = 1000--it will probably diverge, and quickly. Now the fun can begin. Where is the crossover point? Can you determine the crossover value to the precision level of your system? How will you go about it? Which step in the ring is the critical one; that is, the step that determines whether the process will converge or diverge? Does your system converge to 57.5806879... after the "Add 1" step? Does the crossover value change when you improve (or degrade) the cube root step?

PROBLEM 231

The absolute value operations in Rosie's Ring are inserted to insure that the process will survive through the square root step. Is there some other set of eight simple operations that could be performed with the same effects, but that would not involve absolute value operations?

In other words, these Ring algorithms involve something that can be done with any computer (or even a programmable calculator) to explore the world of numbers to some depth.

Exploring Random Behavior--5

PROBLEM 232

Points are selected at random in a square of side 100 units. If any point falls within 10 units of a previously selected point, then both of them are to be deleted from the pattern (and this process could delete as many as five points at once).

As more and more points are selected, this procedure will sort of converge, in the sense that new points will be added at the same general rate as old points disappear.

When that happens, how many points will there be, and what will the pattern look like?

SUPERSTITION

Hedda Hopper Feb 2 1966
Buster Keaton Feb 2 1966
Elizabeth Patterson Feb 1 1966

J. Carroll Naish Jan 24 1973
Edward G. Robinson Jan 26 1973
John Banner Jan 28 1973

Elvis Presley Aug 16 1977
Groucho Marx Aug 20 1977
Sebastian Cabot Aug 23 1977

Richard Conte Apr 16 1975
Marjorie Main Apr 14 1975
Larry Parks Apr 14 1975
Fredric March Apr 13 1975

Inger Stevens
Anita Louise
Gypsy Rose Lee
Ed Begley
April 22-29 1970

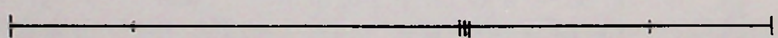
Betty Grable Jul 6 1973
Joe E. Brown Jul 7 1973
Veronica Lake Jul 8 1973
Robert Ryan Jul 11 1973
Lon Chaney, Jr. Jul 13 1973

Rosalind Russell Nov 28 1976
Judith Lowery Nov 29 1976
Godfrey Cambridge Nov 29 1976

Listed above are some sets of show business deaths. There is an old superstition in show business that people die by threes.

They certainly do. Random events must cluster, or they wouldn't be random. We can demonstrate this fact with our computer.

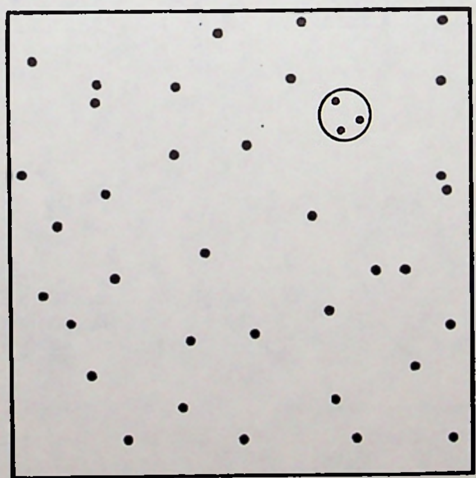
Given a random number generator that outputs numbers uniformly in the range from zero to one. Let a line of length <u>one</u> represent a year's time:



Then each random number can represent a point in time in the one year span. If the random events spaced themselves out in an orderly fashion, then 52 of them could be spaced so that no more than two of them would occur in a space of .01923 (= 1/52); that is, in a space of one week. A 53rd event would have to fall within that range; that is, so that some three events would occur within one week. Actually, if the events are random, this phenomenon will occur much sooner. The most probable number of events is 19. That is, of 19 events occurring at random within a year, some three will usually fall within the space of a week.

We can extend this notion to two dimensions. Select points at random in a unit square. How many points must be selected, on the average, to obtain a cluster of three points that can be enclosed in a circle of diameter .1? See the Figure.

For any three points that appear to form a small cluster, a circle can be passed through them, and its diameter compared to .1. The trick, computationally, is to determine, as each new point is selected, just which of the previously selected points could be considered close enough to the new point to be considered a cluster.



Magic Prime Squares

The following comes from Prof. Richard Andree, one of our contributing editors.

The following problem is suggested for your investigation. A computer may be helpful in the investigation, but the use of (un)common sense (that is, mathematical skill) is also quite desirable.

There are many arrangements of digits into the 16 cells of a 4 by 4 array such that each row and each column contains a four-digit prime number. Some such arrangements also contain primes on the two diagonals.

6	1	1	3
3	0	0	1
1	4	0	9
1	9	9	7

2	1	3	1
1	0	9	1
1	0	0	9
1	9	7	9

(Note: 3041, 6007, 1901, and 1091 are primes. 2009 and 1403 are not primes.)

Your problem is to investigate the possibility of finding one or more 5 by 5 squares which contain five-digit primes in each row, each column, and on the two diagonals.

Measuring Time

BY THOMAS R PARKIN

We all know exactly what is meant by the date. Why, of course, the date is...(and as I sit here writing this, it is Thursday, 7/7/77, a date the likes of which will not repeat for 11 years, 1 month, and 1 day, or for a century, if you insist on a precise repeat)--and we all can say exactly what day of the week it is, what day of the month it is, what month it is, and of what year. Furthermore, I suspect that most of us have a vague notion that the year is related to our earthly rotation about the sun and that the day corresponds to the rotation of the earth about its own axis. And, of course, these are related--365 days equal one year, that is, except for leap years, when somehow we have 366 days---

But how did all this get started and just how is it that we reckon time? For, of course, our days and years and dates are just a way of reckoning elapsed time to our mutual satisfaction and convenience.

Certain primitive people speak of elapsed time as "so many sleeps ago" implying that one sleeps each night and thus marks the passage of "so many days." Likewise, the phrase "so many moons ago" crops up in primitive cultures, implying so many periods of the moon in its approximately 28-day rotation about the earth. It has also been said that "so and so has 73 summers" or "39 winters," to mark passages of years.

Man has probably had some strong feeling for elapsed time and some metrics for measuring it since man has been in existence and, certainly, the durnal rhythm of the earth in its daily rotation is deeply imbedded in the very cellular being of man.

Some of the very earliest techniques for indicating just when it is--are we nearing the harvest time?--are we ready to begin spring planting?--have we come around to that time of year when we should offer a virgin in sacrifice?--began with attempts to mark just when the sun reached its maximum excursion to the North or the South as it rises over the horizon. Man early recognized the correlation between those extremes of sun position and the changes of seasons related to growing crops. The stone monuments in a dozen cultures are arranged so that sighting along some obvious line of two stones, or along some wall or tunnel, toward the horizon, marks the limit of the sun's excursion in some direction.

As it began to be interesting, or important for some purpose, to measure previous time and to be able to refer to it, people devised all manner of ingenious schemes. The ancient Egyptians used the method of referring to past time as "such and such a year in the reign of Pharaoh X." Since most time references were with respect to a person's life span, it was only natural to use the chief's life to measure an era. Many cultures used this scheme. Other cultures invented more or less elaborate means of counting days and years in which they measured from some real or imaginary singular event in their remembered or legendary history.

But whatever the means, man has measured time almost since time began, and it is not just now that we have been so smug in knowing just "what the date is." However, it <u>is</u> only comparatively recently that we have begun to be concerned with time spans which exceed a single, or at most a few people-lifetimes, and, particularly, since the advent of astronomy as a science. We shall see how this interest is accommodated shortly. But first, let us look a bit at the calendar.

The first calendar in modern form was decreed by Julius Caesar in what is now known as 46 (or sometimes 45) B.C. (There is some confusion about this and we shall see the reason shortly.)

The preceeding year, known as "the year of confusion," was extended to 445 days in order to bring the equinoxes and solstices to the dates which were felt to be proper; the months had their names and lengths changed; and general confusion reigned as to exactly "what is the date today?"! The year Caesar decreed what is now called the Julian Calendar was the year 703 AUC by Roman reckoning; that is, 703 years after the legendary founding of the City of Rome.

The next significant change in counting time occurred in the sixth century when a Roman monk named Dionysius Exiguus suggested that the dating of history begin with the birth of Christ. He reasoned that Christ had been born December 25, 753 AUC (by the Roman Calendar) and he proposed that the start of the Christian era be January 1, 754 AUC. It was at this point, and forever more, that the mistake in counting took place. 754 AUC became year 1 of the Christian era, not zero A.D. Look at it this way: suppose we have a scale marked off in years, with AUC numbers and A.D. numbers, the way they are commonly understood:

751	752	753	754	755	756	AUC dates
Z	Y	X	1	2	3	Christian era dates

(Supposed) Birth of Christ

What should we call years X, Y, and Z?

Unfortunately, X is called 1 B.C. or the first year before the birth of Christ; Y is 2 B.C. and Z is 3 B.C. Thus we are stuck with no year zero. (Indeed, the year 2000 is the last year of the 20th century, and the 21st century doesn't start until January 1, 2001.) This lack of a year zero leads to endless confusion when differencing dates across that boundary. Everyone keeps being surprised at some one year discrepancy which keeps cropping up. Blame Dionysius! The primary error was his in calling 754 AUC the year one A.D.

In any event, the idea spread slowly from what Dionysius calculated was 532 A.D. until its first adoption by an entire country in 879 A.D. by Emperor Charles III of Germany. By around 1000 A.D. the use of the Christian numbering of the years had become widespread in the Western world. To this day it is not universal, however. Several other year counting schemes exist; the Chinese have one, the Arabs another, and the Jews still another. Nevertheless, there is reasonably good agreement throughout the civilized world today just what the date is and the most widely accepted numbering scheme is the one based on the Christian era, regardless of what may be the local ecclesiastical numbering scheme.

Julius Caesar's contribution to the calendar was to decree, with the advice of the Greek astronomer and mathematician Sosigenes, that the mean solar year was 365 1/4 days long and that after 3 years of 365 days each there would be a leap year of 366 days. This was about the same time that the 12 months with their varying numbers of days became fixed, and the extra or intercalary day was given to February because it was the shortest month!

The implication of a 365 1/4 day mean year is actually in error by about 11 minutes and 14 seconds too long. The current best data on the length of the mean solar year is 365 days, 5 hours, 48 minutes, 45.6170 seconds (as of January 1, 1975, according to the Naval Observatory, and slowing down by .53 seconds for each century after January, 1900) or 365.2421946412 days. For purposes of average calculation, however, the commonly accepted 11 minutes, 14 seconds less than 365 1/4 days per year is quite satisfactory. Indeed, in the lifetimes of a few people, this difference isn't really noticeable and the Julian Calendar enjoyed widespread acceptance for quite a long time--that is, apart from the numbering of the years, as we have seen.

However, by the 16th century, the error in accumulated time between the mean solar year and the Julian calendar had become 10 days and the vernal equinox had slid back to March 11 instead of the presumed correct date of March 21, where it was in 325 A.D. when the rule for determining Easter was formulated at the Council of Nicaea (for details of that calculation, see our issue No. 13). Pope Gregory XIII decreed then that the day after October 4, 1582 should be called October 15, 1582, thus dropping ten days.

Furthermore, he announced the continuing rule that henceforth every century year except those evenly divisible by 400 would <u>not</u> be a leap year, thus <u>almost</u> correcting the problem. This new rule, combined with the normal leap year rule of every 4 years then allows the astronomical solar year and the ecclesiastical calendar year (which has since become our generally accepted civil year) to stay less than one day apart for over 3300 years.

Another rule has been suggested to the effect that every 4000 years the year be an ordinary year even though Pope Gregory's rule would make it a leap year. This correction would then be good for several hundred centuries!

Not too many people are really concerned with calendar printer's problems 2000 years from now, so this new 4000 year rule isn't a hot item, but it has been proposed.

So now we come to Joseph Scaliger, who also did something about the calendar in 1582. By then, astronomers were interested in dating events as exactly "so many" days apart. With all the confusion about dates and all the very many calendars in use about the world, some useful means of identifying particular days would be thankfully accepted. What he did was to assign a serial number to a day according to a cycle which started on January 1, 4713 B.C. and which runs for 7980 years. This is called a Julian period, and the serial number of the day in that cycle is called a Julian day number. Thus, the Julian day numbers of two astronomical events being known, the elapsed time between the events is obtained by a single subtraction and no worry about which calendars were used or how many leap years there were, etc.

Scaliger determined his starting date by an intricate process. In addition to all the other confusion about the calendar, there were (and still are) three cycles of elapsed years which were important: The Roman indiction cycle of 15 years; the solar cycle of 28 years; and the lunar cycle of 19 years. Scaliger composed what he called a Julian period of 7980 years by taking the least common multiple of 15 x 28 x 19 = 7980.

The 28 year solar cycle derives from all possible combinations of days of the week with the start of the year, allowing for leap years. The 19 year lunar cycle comes from the observation that 19 solar years very nearly (within 2 hours) equals 235 lunar months and the phases of the moon repeat themselves on the same dates every 19 years, with a possible one day difference depending on the number of leap years in the particular 19 year span.

The 15 year Roman indiction cycle refers to the periodic valuation of property for tax purposes (wouldn't it be just great if <u>our</u> tax assessors came around only every 15 years!) instituted in the fourth century A.D. The American Ephemeris and Nautical Almanac still gives the values of these three cycles for any given year!

The starting point or epoch for any of these cycles can be arbitrary, but once started, by common oonvention, the current year is uniquely determined, mod 7980, by knowing the year number in each of the three cycles. Thus, the year 1582 A.D. was 10 in the indiction cycle, 23 in the solar cycle, and 6 in the lunar cycle. By running these cycles ahead until each of the three ends at the same year, we will be at the year 3267 A.D. If we subtract the full Julian cycle of 7980 years from that date, we get 4713 B.C. as the beginning of the Julian period. Thus we have that the Julian period began on 1 January 4713 B.C. and the next one will start on 1 January 3268 A.D.--this sums to 7981 because of that lack of a year zero, as we have noted before--and the length of the Julian period is, according to Scaliger, 365 1/4 x 7980 = 2,914,695 days. This is a period of 60 days more than there will elapse from 1 January 4713 B.C. to 1 January 3268 A.D., and we shall see how <u>this</u> comes about!

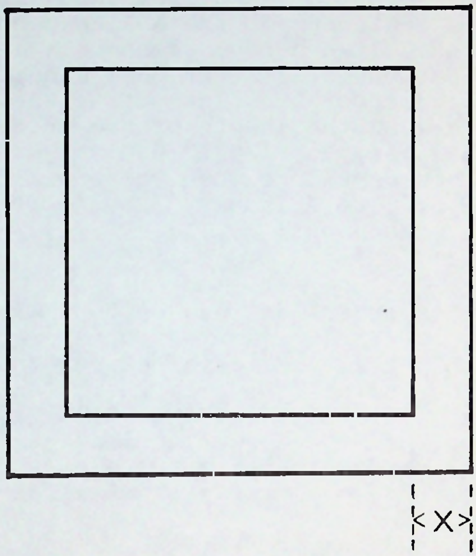
The Gregorian calendar, decreed by Pope Gregory XIII, with its elimination of three leap year days every 400 years, will yield only 2,914,635 days between the start of the current Julian period and the arbitrary start of the next one on 1 January 3268 A.D. This will give no trouble to any of us, but the Gregorian leap year adjustment <u>does</u> give us trouble when we attempt to calculate exactly what Julian day number we have for any given date.

The Gregorian calendar was adopted by Pope Gregory in 1582. England and the American colonies switched in 1752, eliminating September 3 through 12 (11 days) that year; Japan switched in 1873, China in 1911, Greece in 1924, and Turkey in 1927. The various times for the change caused differing numbers of days to be subtracted, since the Julian calendar relentlessly kept picking up days--3 every 400 years. This should give us pause to consider--just what <u>do</u> we mean by enunciating some given date? When we ask for the Julian day number for January 29, 1066 A.D., what is our reference? Turkey in 1926? California in 1977? London in 1620? There is a way we can eliminate some confusion and that is by arbitrary agreement. We shall see how this comes about.

At the time Joseph Scaliger proposed the Julian day number scheme (JD or JDN) in 1582, the Julian calendar was all he really knew about; hence his JDN was with reference to <u>that</u> calendar and <u>not</u> to the Gregorian calendar. His epoch of 1/1/4713 B.C. gave that day a JDN of 1. Then, by the Julian calendar, each next 100 years had exactly 25 leap years and 75 regular years, for a total of 36525 days, right up to October 4, 1582, which has a JDN of XX,XXX,XXX. (Remember, 4713 B.C. is year -4712 because 1 B.C. is year zero.) (A short table of a few key JDN's appears at the end of this article.) □

(Part one. Part two will appear in a subsequent issue.)

Stripping Away



PROBLEM **235**

Start with a square of side 100 units. We take away 100 square units by removing a strip of width x (.250628 units) from all four sides of the original square. This leaves a new smaller square, of side 99.498743 units. The length of the strip that was removed is 398.99748 units.

We now repeat the process, only this time removing 99 square units. The width of the second strip will be .2493718 units, and its length will be 396.997488 units.

The third strip takes away 98 square units...and so on, until the 100th strip removes one square unit.

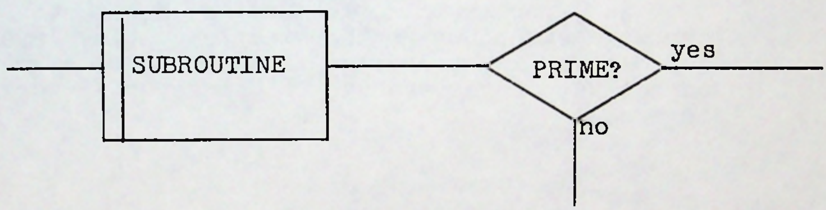
Problem: What is the total length of the 100 removed strips?

homework 2

1009
1013 *
1019 *
1021
1031
1033 *
1039 *
1049
1051
1061
1063 *
1069 *
1087 *
1091 *
1093 *
1097 *
1103 *
1109 *
1117 *
1123 *
1129 *
1151
1153
1163
1171
1181 *
1187 *
1193 *
1201
1213
1217 *
1223 *
1229 *
1231 *
1237 *
1249
1259
1277 *
1279
1283 *
1289 *

The list shown at the left is of consecutive prime numbers. The numbers marked * are those primes, p, for which p+6 is also a prime.

Assume that we have a subroutine that can certify whether or not a given number is prime:



We wish to search the range of integers from 1001 through 9999 and sum those for which the difference between them is 6 (this is the most frequent difference between primes); that is, sum all those numbers that would be marked (*) as on the sample list.

Notice that it is not even necessary to know what a prime number is. With the stated subroutine available, we have the means of producing the required sum.

Draw a flowchart of the logic involved in this problem.

Due date:____________________

Outline a procedure to test a debugged program that follows that logic.

Due date:____________________

Note: The sum involved in this exercise is 3060480, but the problem is the logic of how to get that sum efficiently.

Suppose we classify any given integer on the basis of the remainder on division of that integer by 4. We can thus create four classes:

Class A--those integers exactly divisible by 4.

Class B--those having a remainder of 1 on division by 4.

Class C--those having a remainder of 2 on division by 4.

Class D--those having a remainder of 3 on division by 4.

For example, 65744 is of class A, 12377 is of class B, 238 is of class C, and 98711 is of class D.

There are 64 integers stored in a block of words addressed at T, T+1, T+2,...,T+63. We want to move them around in the block so that all the class A numbers come first, then all the class B numbers, then all the class C numbers, and finally all the class D numbers. It would be nice also if the original ordering within each class was preserved, but this is not vital.

See, for example, the initial arrangement and the rearrangement shown in the example. (The layout in an 8 by 8 array is of no significance; it is simply a compact way of representing 64 consecutive words of storage.)

Mod 4

51	35	22	7	70	49	28	6
79	37	72	63	23	69	75	13
12	47	3	46	61	2	41	67
78	21	64	333	59	89	14	42
27	73	85	9	32	56	29	66
93	74	20	81	1	43	76	45
4	39	87	83	777	54	15	8
65	26	50	19	34	53	25	52

Source data--more or less random integers.

28	72	12	64	32	56	20	76
4	8	52	49	37	69	13	61
41	21	33	89	73	85	9	29
93	81	1	45	77	65	53	25
22	70	6	46	2	78	14	42
66	74	54	26	50	34	51	35
7	79	63	23	75	47	3	67
59	27	43	39	87	83	15	19

The same data rearranged mod 4.

Several schemes suggest themselves for this task:

1. Examine all the numbers in block T and move them to temporary blocks (each of length 64 words) A', B', C', and D'. Then move the contents of the temporary blocks back to block T in the required order.

2. Assume a 32-bit word machine, and the given data is such that no number exceeds, say, 20 bits in length. Analyze each number to determine its class and then (assuming only positive numbers);

for class A, do nothing;

for class B, add 2^{25};

for class C, add 2^{26};

for class D, add 2^{27}.

Now, sort the numbers in block T (after which, the extraneous additives should be removed).

3. First, go through and count the size of each of the four classes, to establish the starting addresses of the final four sub-blocks. Set up pointers for these addresses. Start with the first word of sub-block A. If the number stored there is of class A, go on. If it is not, then store it in the proper sub-block, thus displacing some other number, unless that number is in its proper sub-block.

This procedure is best explained by example. Using the data given, the preliminary count shows the following:

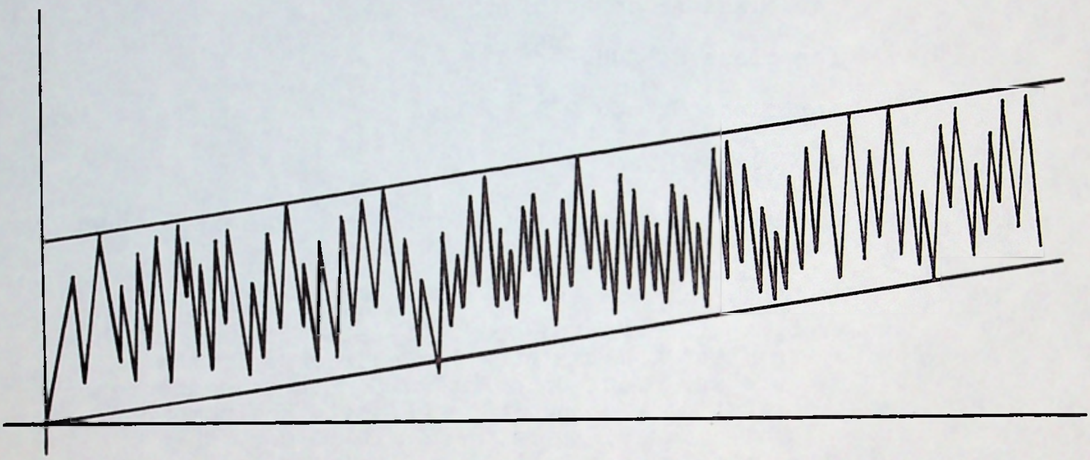
11 of class A
21 of class B
14 of class C
18 of class D.

So the leading addresses of the four sub-blocks are T, T+11, T+32, and T+46. We start with the first word of sub-block A, which contains 51, which is of class D. The 51 is thus to move to T+46, where it displaces 76. The 76 is of class A and can be moved to address T. The next word to be considered is T+1, which contains 35, also of class D. The 35 moves to T+47, displacing 45 which moves to T+11, displacing 63 (class D) which moves to T+48...and so on until all 64 numbers have been examined and/or moved. Offhand, it sounds as though method 3 would be the most efficient (in terms of machine time) but would be the stickiest to code, in any language.

As we said, these three schemes leap to mind right away. Surely there is a better way...

Fabricating A Sequence

Let's trace the evolution of a new sequence. To start with, what is wanted is a succession of values, V, as a function of the term number, N, which will show a growth pattern something like this:



Thus, the values of V should increase steadily, with much local perturbation, occasional peak values, and no discontinuities.

So try 1 is this: let V go up by 1 when N is prime, and down by 1 when N is composite. This is clearly no good; the composites will far outnumber the primes, and the trend will be sharply negative.

Try 2: suppose we adjust the weights, letting V go up by K for each prime, and down by 1 for each composite. The ratio of primes to composites becomes fairly stable when N is large, but no value of K will produce a pleasing sequence when N is small. Moreover, such a sequence will be dealing with large numbers for V rather quickly.

If V is to be determined by the nature of N, then only odd values of N should be considered in the sequence.

Try 3: Suppose we increase V by (p-1)/2 for each prime value of N and decrease V by 1, 3, 5, 7,... for successive composite values of N. It's not bad, but it yields a dull sequence.

Try 4: To keep the values small, let's use multiples of

$$\left[\sqrt{p}\right] \quad \text{(the greatest integer in the square root of p)}$$

as our additive component. Call this quantity D.

Suppose we add 2D for every prime value of N and subtract (D+1) for every composite value of N? This also isn't too bad; it has some attractive features. Eventually, however (around N = 400) it goes negative and just keeps getting more negative.

Maybe the thing to do is to prevent V from going negative. How about this: let V increase by D for every prime value of N. For composite values of N, decrease V by (D+1) unless that makes V negative, in which case increase V by (D+1).

This is better, except that when a long string of composites comes along, the sequence may simply oscillate. If V is 15, for example, and (D+1) is 16, then for a long string of composites, the sequence will run

31, 15, 31, 15, 31,...

and this is dull.

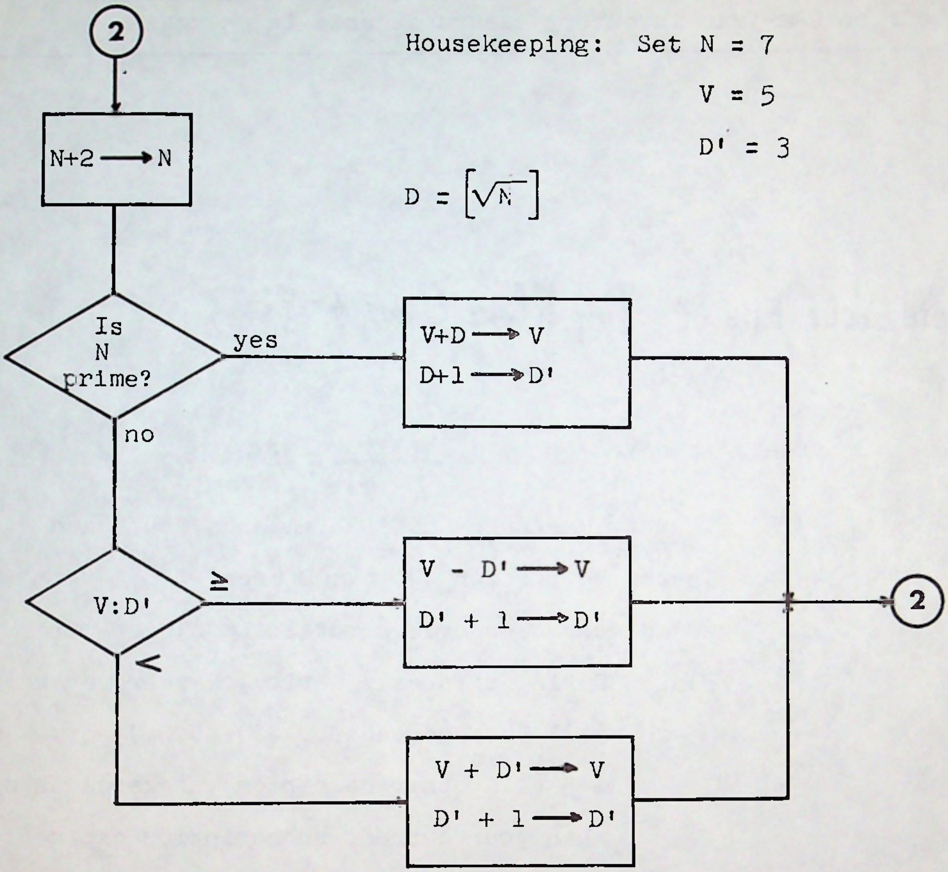
Try 5: Let's let the increment itself vary. We'll have V increase by D for every prime value of N, and set D' = D + 1 at that time. For each composite value of N, then, we will attempt to subtract D' unless that rolls negative; otherwise add D'. And, for each composite, D' itself will increase by 1. This begins to look like an interesting sequence. The first 100 terms are these:

1	0	41	16	81	6	121	21	161	11
3	1	43	22	83	15	123	6	163	23
5	3	45	15	85	5	125	22	165	10
7	5	47	21	87	16	127	33	167	22
9	2	49	14	89	25	129	21	169	9
11	5	51	6	91	15	131	32	171	23
13	8	53	13	93	4	133	20	173	36
15	4	55	5	95	16	135	7	175	22
17	8	57	14	97	25	137	18	177	7
19	12	59	21	99	15	139	29	179	20
21	7	61	28	101	25	141	17	181	33
23	11	63	20	103	35	143	4	183	19
25	6	65	11	105	24	145	18	185	4
27	0	67	19	107	34	147	3	187	20
29	5	69	10	109	44	149	15	189	3
31	10	71	18	111	33	151	27	191	16
33	4	73	26	113	43	153	14	193	29
35	11	75	17	115	32	155	0	195	15
37	17	77	7	117	20	157	12	197	29
39	10	79	15	119	7	159	25	199	43

The following information about this sequence:

N	new larger V	Zeros	Ratio V/N
27		0	
37	17		
43	22		
61	28		
103	35		
109	44		
155		0	
223	45		.20179372
229	60		
279		0	
421	70		.16627078
447		0	
463	85		
601	98		.16306156
703		0	
811	100		.12330456
829	125		
849		0	
1047		0	
1063	130		.12229539
1391		0	
1411		0	
1483	139		.09372893
1489	176		
1565		0	
1719		0	
2025		0	
2319		0	
2367		0	
2713	206		.07593070
3009		0	
3259	253		
3431		0	
3469	292		

is to be extended, showing the appearance of each new larger value of V, each zero value of V, and each lower value of the ratio of V/N, taken whenever a new larger value of V appears.



The logic for extending the sequence described above. Since the logic is very simple, the sequence can be extended readily on nearly any computer, or on a programmable calculator. Let's call it sequence S.

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